Quantum Filtering for Optimization and Tomography of Nonlocal States (QFOTONS)

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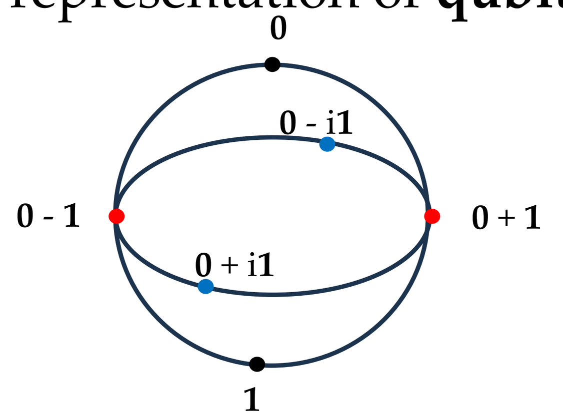
Introduction

- Quantum information has widespread applications in **communications, imaging, and encryption.**
- Basic representation of **qubit**:

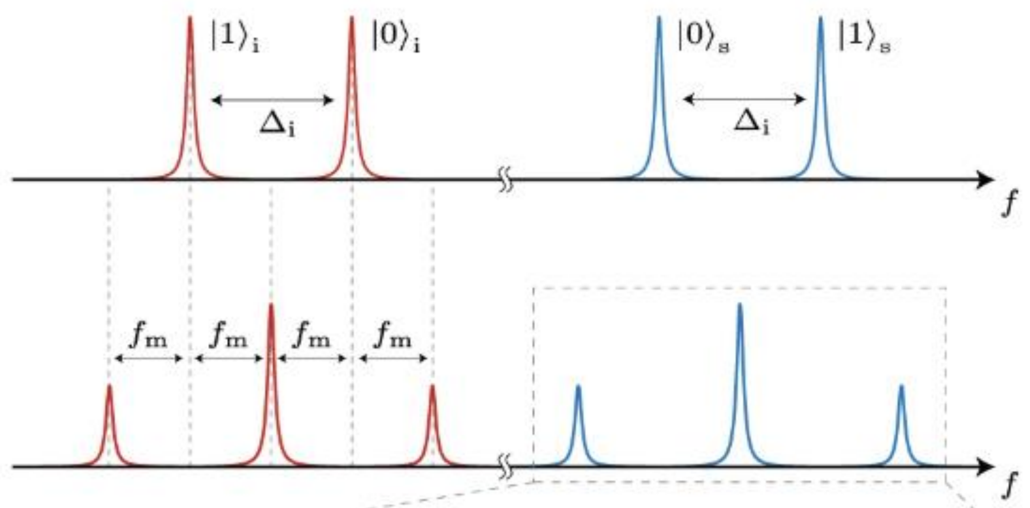


- For higher dimensions, we can represent quantum states as **qudits**, represented by a **density matrix**

$$\rho = \sum_{i=1}^{D}\sum_{j=1}^{D} a_{ij}|i\rangle\langle j|,$$

Requirements:
1. Unit trace: For density matrix ρ, Tr(ρ)=1
2. Hermitian: $\rho^{\dagger}=\rho$
3. Positive semi-definite: $\langle\psi|\rho|\psi\rangle\geq 0$

- We aim to transfer quantum information through frequency bins.

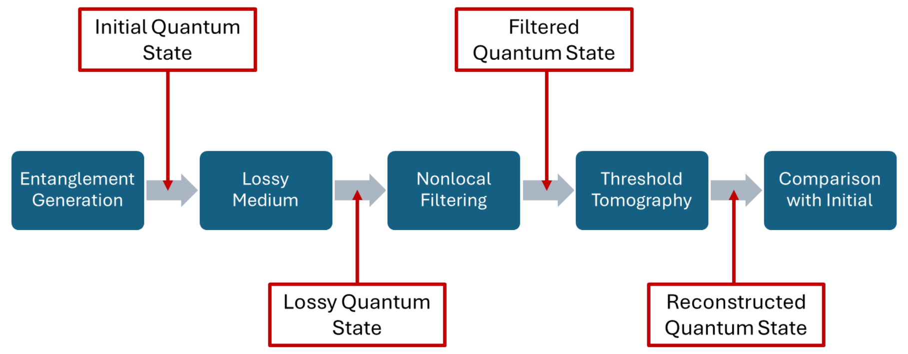


Clementi, M., Sabattoli, F.A., Borghi, M. *et al.* Programmable frequency-bin quantum states in a nano-engineered silicon device. *Nat Commun* **14**, 176 (2023). https://doi.org/10.1038/s41467-022-35773-6

This project aims to:
1. develop a numerical scheme for **recovering photon entanglement** between frequency bins due to losses in certain bins for high dimensional photons
2. develop a statistical robust scheme for **reduces the number of measurements** needed.

MATLAB SIMULATIONS

Nonlocal Filtering



Here, we compared the **log negativity** of three projectors:

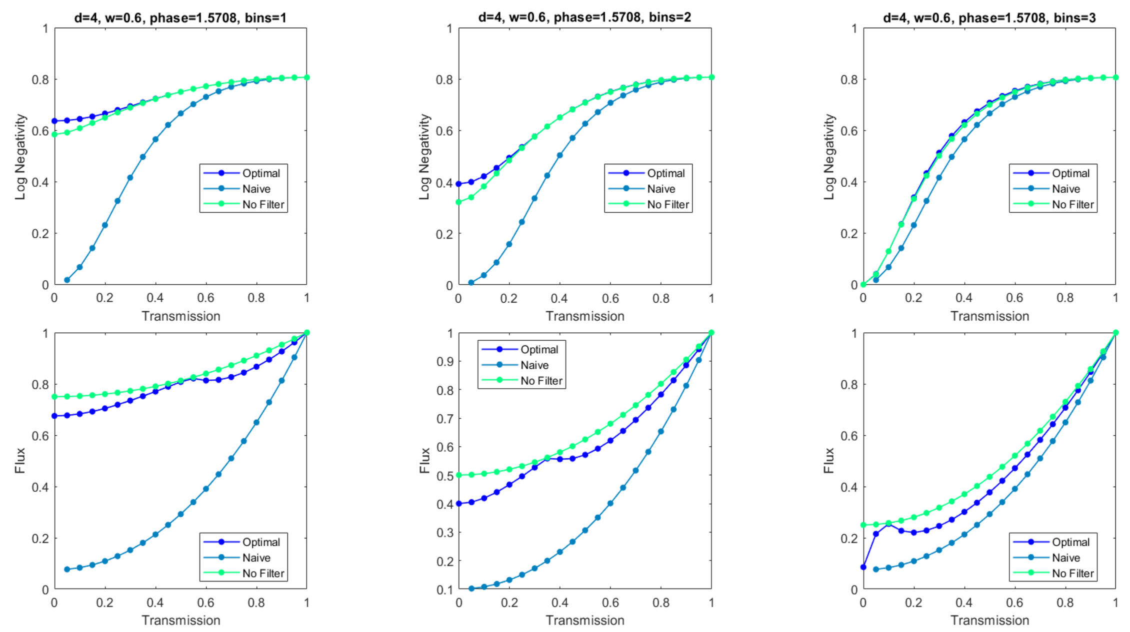
1. identity (No Filter)
2. an inverse (Naive)
3. a numerically solved filter (Optimal)

$$E_N(\rho_w) = 0, for\ \lambda \leq \frac{1}{d+1}$$

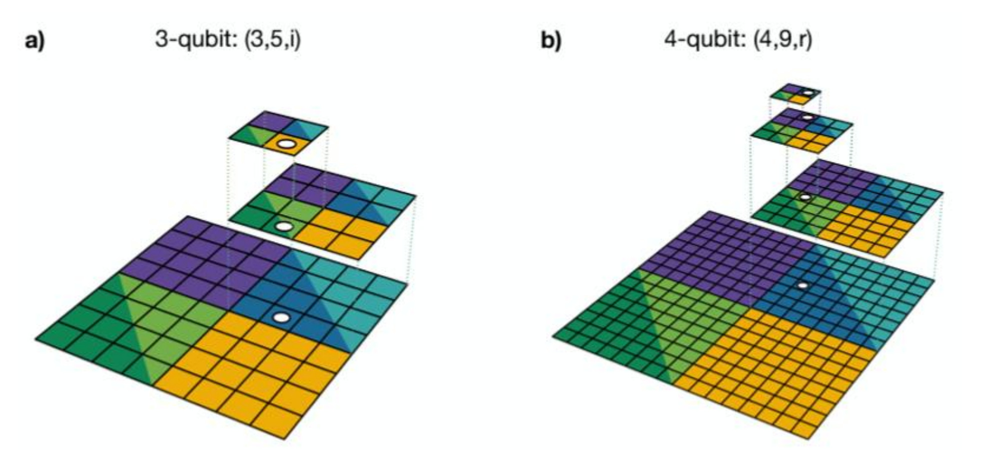
$$E_N(\rho_w) = log_2\frac{1+\lambda(d^2-1)}{d}, for\ \lambda > \frac{1}{d+1}$$

We filtered losses in the state to construct a maximally entangled state by applying projectors on the density matrix.

Our conclusion was to use No Filter as it maintained the most entanglement without losing as much flux (number of photons)



Threshold Tomography



D. Binosi, G. Garberoglio, D. Maragnano, M. Dapor, and M. Liscidini, "A tailor-made quantum state tomography approach," *Deleted Journal*, vol. 1, no. 3, Jul. 2024, doi: https://doi.org/10.1063/5.0219143.

Using Cauchy-Schwarz Inequality, we can find that off diagonal elements have the property:

$$\rho_{i,j} \leq \sqrt{\rho_i\rho_j}$$

We can set a threshold based on measurements on the diagonal of the matrix:

$$t = \sqrt{\rho_i\rho_j}$$

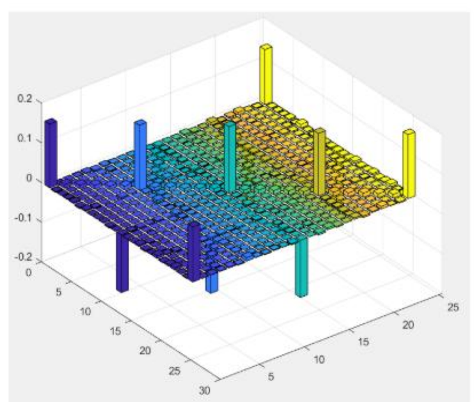
$$\rho_\phi = \sum_{m,n}^{d} i^{m+n}|mm\rangle\langle nn| \quad \rho_w = \frac{w}{d}\rho_\phi + \frac{1-w}{D}\mathbb{I}$$

$$\mathcal{F} = 0.9738$$

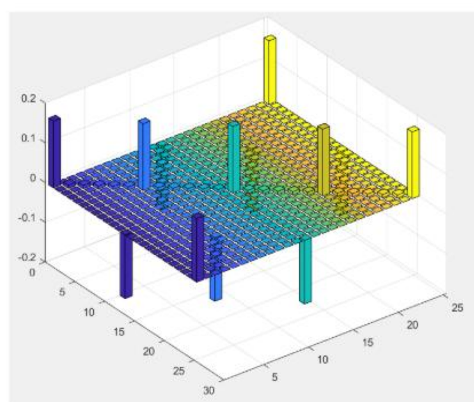
We use projectors:

$$|X_{ab}X_{cd}\rangle = \frac{1}{4}(|a\rangle+|b\rangle)(|c\rangle+|d\rangle)$$
$$|X_{ab}Y_{cd}\rangle = \frac{1}{4}(|a\rangle+|b\rangle)(|c\rangle+i|d\rangle)$$

Using Bayesian inference, we construct states with a fidelity of about 0.97, which is acceptable for the described relevant applications.

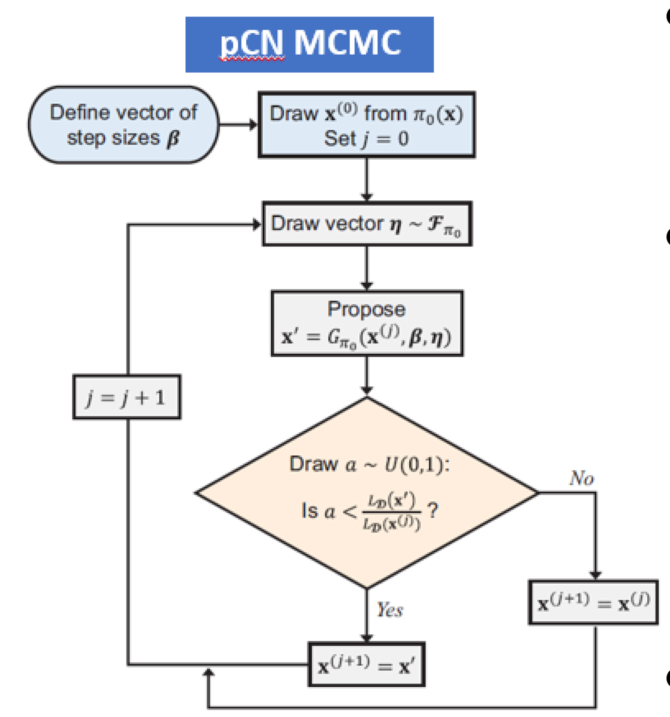


Reconstructed state



Ground state

BAYESIAN INFERENCE

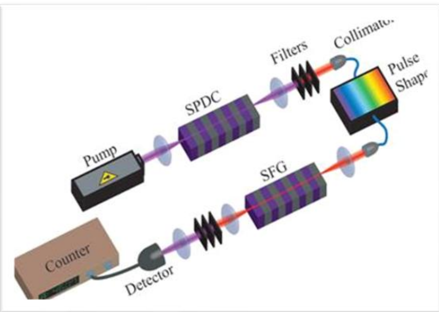


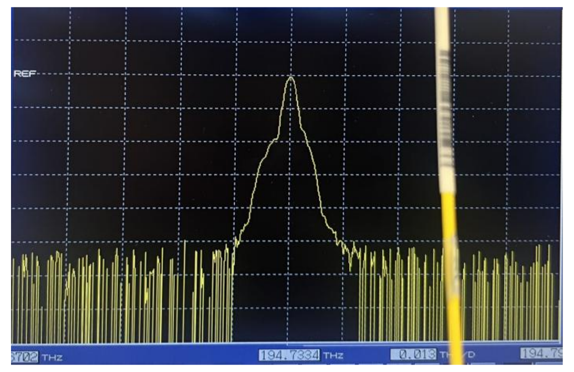
J. M. Lukens & A. Passian, *Phys. Rev. A* **104**, 053501 (2021).

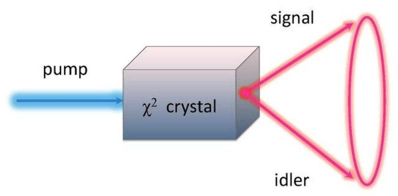
- Random walk, likelihood comparison to obtain probability distribution
- MCMC: for an unknown probability distribution, we can artificially "draw samples" assuming that we're able to sample from the probability distribution
- We save these samples do that we can make estimations

EXPERIMENTAL IMPLEMENTATION

- Infrared 780 nm laser for the generation of entangled photons







- Periodically poled lithium niobate wave guide generates entangled biphoton through SPDC
- Electro-optic modulators and pulse shapers for state projections

RESULTS AND CONCLUSION

- Though exponential threshold tomography simulations allowed for reduced measurements compared to a full measurement scheme
- No filter maintained the most entanglement compared to flux loss in nonlocal filtering
- Bayesian inference method allowed for a full characterization of quantum states in an efficient time